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BIOGRAPHY.

BENJAMIN PEIRCE.

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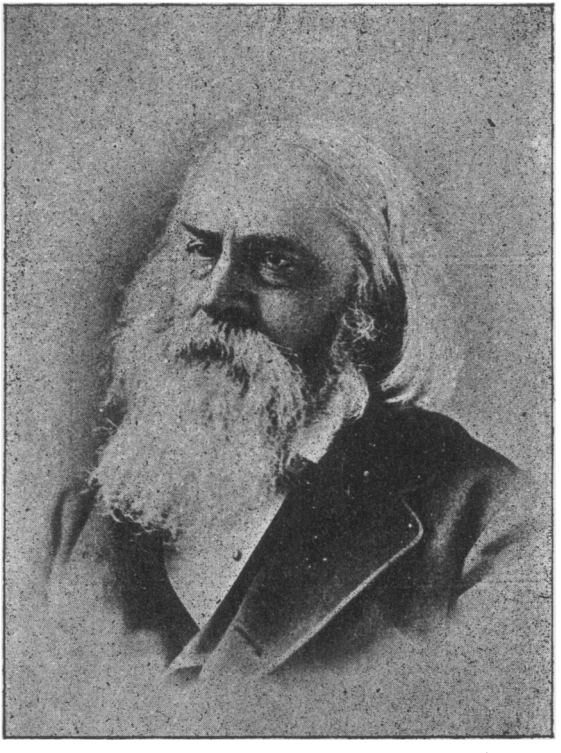
BENJAMIN PEIRCE was born at Salem, Massachusetts, April 4, 1809, and died at Cambridge, Massachusetts, October 6, 1880. He entered Harvard College, at the age of sixteen; and, at the age of twenty, he was graduated from the same College, with highest honors. He devoted himself principally to the study of Mathematics. This *favorite* study of his was pursued far beyond the limits of the curriculum of mathematical studies prescribed by the authorities of Harvard College, at that time.

As an under-graduate student, young Peirce was instructed by Nathaniel Bowditch, who soon perceived the innate mathematical genius of his pupil. Bowditch proudly predicted the future greatness of the young man. Not only did Bowditch give him valuable instruction in geometry and analytics, but also acted as his *mathematical adviser*—carefully directing him in the development of his mathematical talents and scientific powers. The lectures on higher mathematics delivered by Francis Grund he was enabled to attend, by reason of his preparation beyond the limit of the under-graduate course in mathematics. When Dr. Bowditch was publishing his translation and commentary of the *Mehanique Celeste* of Laplace, young Peirce assisted in reading the proof-sheets. This critical reading of that great work of Laplace was to him an education in itself, and may have been the prime cause that not a small part of Peirce's subsequent mathematical and scientific work was done in the great field of analytical mechanics:

In the class-room, he frequently gave original demonstrations which



BENJAMIN PEIRCE.

proved to be more direct and scientific than those given in the text-books of that day. On graduating, he went to Northampton, Massachusetts, as a teacher in Mr. Bancroft's School. As *tutor*, he returned to Harvard College, in 1831. Since Professor Farrar spent the next year in Europe, tutor Peirce was left at the *head* of the Department of Mathematics in Harvard College; and, on account of the physical inability of Professor Farrar to resume teaching, Peirce *continued* to fill his place. In fact, Peirce held this position, advancing step by step, until the time of his death. His position, in 1842, was christened "The Perkins Professorship of Mathematics and Astronomy." In the history of mathematical teaching at Harvard College, the year 1833 marks an important epoch; as it was then that Benjamin Peirce became the *professor* of Mathematics and Natural Philosophy in that institution of learning.

Professor Peirce was married in July, 1833. At the time of his death, there were living his wife, three sons, and a daughter. His eldest son, James M. Peirce, is University professor of mathematics in Harvard; Charles S Peirce is a professor in the Johns Hopkins University; and H. H. D. Peirce is connected with the firm of Herter Brothers, New York City.

It has been said that a mere boy detected an error in Bowditch's solution of a problem. "Bring me the boy who corrects my mathematics," said Bowditch. Master Benjamin Peirce was the boy who had done the correcting; and thirty years later, this same Benjamin Peirce dedicated one of his great mathematical works "To the cherished and revered memory of my master in science, Nathaniel Bowditch, *The Father of American Geometry.*" This same title was bestowed upon Peirce, by foreign mathematicians. Sir Wm. Thomson (Lord Kelvin), in an address before the British Association, referred to Benjamin Peirce as "*The Founder of High Mathematics in America*"; and on a similar occasion, the late Professor Cayley referred to him as "*The Father of American Mathematics.*" The name of Benjamin Peirce is that of an *American mathematician*, whom no one need hesitate to rank with the names of Pythagoras, Leibnitz, Newton, Legendre, John Bernoulli, Wallis, Abel, Laplace, Lagrange, and Euler. Through the united efforts of the late Professor Wm. Chauvenet (Yale's ablest mathematician and astronomer) and Benjamin Peirce —not to speak of their worthy successors, was effected the general adoption of the *ratio-system* in American works on trigonometry.

In the reforms incident to the *New Education*, Harvard has always taken a prominent part and Benjamin Peirce was *an enthusiastic advocate* of the elective system with respect to collegiate studies. As a branch of Harvard College, there was opened, in 1842, the Lawrence Scientific School; and in this school, Professor Peirce gave instruction in higher Mathematics including analytical and celestial mechanics. Such advanced courses of mathematics, as *he* offered to students, in 1848, had never before been offered to American students by any other professor in any other American college. The second American educational institution which offered equally advanced courses of mathematics, is the Johns Hopkins University; and these courses were arranged by that *English master*, who gave a fresh and powerful impulse to mathe-

matical study and teaching in America—*Professor J. J. Sylvester.*

The preparation of mathematical text-books was begun by Professor Peirce, immediately on beginning his career as teacher of Mathematics in Harvard College. In 1835 appeared his *Elementary Treatise on Plane Trigonometry*; in 1836, his *Elementary Treatise on Spherical Trigonometry* together with his *Elementary Treatise on Sound*; in 1837, his *Elementary Treatise on Plane and Solid Geometry* together with his *Elementary Treatise on Algebra*; during the period 1841-46, he wrote and published in *two* volumes his *Elementary Treatise on Curves, Functions, and Forces;* and in 1855, he published his *Analytical Mechanics.* Subsequently was published his memoir on *Linear Associative Algebra*; and this memoir, according to Professor James Mills Peirce, he regarded as his great work. All of his works are models of conciseness, perspicuity, and elegance; and they all evince extraordinary originality and genius.

In 1867, Professor Peirce was made the Superintendent of the United States Coast Survey; and he held that position for seven years. He had been consulting astronomer to the *American Ephemeris and Nautical Almanac*, since 1849; and for many years, he *directed* the theoretical part of the work. In 1855, Professor Peirce was one of the men intrusted with the organization of the Dudley Observatory. For many years *before* and *after* he took charge of the United States Coast Survey, he was frequently consulted with respect to the work in that office. He received the degree of *Doctor of Laws* from the University of North Carolina, in 1847, and also from Harvard University in 1867. He was elected an Associate of the Royal Astronomical Society of London in 1849 and a member of the Royal Society of London in 1852. He was elected president of the American Association for the Advancement of Science, in 1853 (the fifth year of its existence); and he was one of the *original* members of the Royal Societies of Edinburg, and Gottingen; Honorary Fellow of the Imperial University of St. Vladimir, at Kiev;etc.

Professor Peirce's conception of the American Social Science Association was that it should be a *university for the people*,—combining those who can contribute any thing original in social science into a temporary academical senate, to meet for some weeks in a given place and debate questions with each other, as well as to give out information for the public. In this line of thought he favored, also,the establishment of the Concord School of Philosophy, to do a similar work in the speculative studies; and he lived to see the partial realization of what he foresaw in this instance. In a Mathematical Society over which he presided for some years, each member would bring something novel in his own particular branch of study; and in the discussion which followed, it would almost invariably appear that Professor Peirce had, while the paper was being read, pushed out the author's methods to far wider results than the author had dreamed possible. The same power of extending rapidly in his own mind novel mathematical researches was exhibited at the sessions of every scientific body at which he chanced to be present. What was quite as admirable was the way in which he did it, giving

the credit of the thought always to the author of the essay under discussion. His pupils thus frequently received credit for what was in reality far beyond their attainment. He robbed himself of fame in two ways: by giving the credit of his discoveries to those who had merely suggested the line of thought, and by neglecting to write out and publish that which he had himself thought out.

In physical astronomy, perhaps, his greatest works were in connection with the planetary theory, his analysis of the Saturnian system, his researches regarding the lunar theory, and the *profound criticism* of the discovery of Neptune following the investigations of Adams and Leverrier. At the time of the publication of his "*System of Analytical Mechanics*," Professor Peirce announced that the volume would be followed by three others, entitled respectively: "*Celestial Mechanics*," "*Potential Physics*," and "*Analytical Morphology.*" These three volumes were never published.

Professor Peirce, in a paper read before the American Association for the Advancement of Science, in 1849, showed in the vegetable world the demonstrable presence of an intellectual plan—showed that phyllotaxis (the science of the relative position of leaves) involved an algebraic idea; and this algebraic ida was subsequently shown to be the solution of a physical problem.

The higher mathematical labors of so eminent a geometer must lie beyond the course of general recognition. Among the things which give him a just claim to this title, may be mentioned: his discussion of the motions of two pendulums attached to a horizontal cord; of the motions of a top; of the fluidity and tides of Saturn's rings; of the forms of fluids enclosed in extensible sacs; of the motions of a sling; of the orbits of Uranus, Neptune, and the comet of 1843; of the criteria for rejecting doubtful observations; of a new form of binary arithmetic, of *systems* of linear and associative algebra; of various mechanical games, puzzles, etc.; of various problems in geodesy; of the lunar tables; of the occultations of the Pleiades; etc. He adapted the epicycles of Hipparchus to the analytical forms of modern science; and he, also, solved by a system of co-ordinates of his own devising, several problems concerning the involutes and evolutes of curves, which would probably have proved impregnable by any other method of mathematical approach.

None of Professor Peirce's labors lie farther above the ordinary reach of thought than his little lithographed volume on Linear and Associative Algebra. In this he discusses the nature of mathematical methods, and the characteristics which are necessary to give novelty and unity to a calculus. Then he passes to a description of seventy or eighty different kinds of simple calculus. Almost no comment is given; but the mathematical reader discovers, as he proceeds, that only *three* species of calculus, having each a unity in itself, have been hitherto used to any great extent,—namely, *ordinary algebra*, *differentials*, and *quaternions*. Think of it; what a wonderful volume of prophecy that is which describes seventy or eighty species of algebra, any one of which would require generation after generation of ordinary mathematicians to develop!

On both sides of the Atlantic, Professor Peirce as an author, was highly esteemed. His work on analytical mechanics was, at the time of its publication, regarded even in Germany, as the *best* of its kind. As a lecturer, Professor Peirce was highly esteemed in both scientific and popular circles. It is related that in 1843, by a series of popular lectures on astronomy, he so excited the public interest that the necessary funds were immediately supplied, for erecting an astronomical observatory at Harvard College. A remarkable series of lectures on "*Ideality in Science*," delivered by him in 1879 before the Lowell Institute in Boston, attracted the general attention of American thinkers, on account of the thoughtful consideration of the vexed question of science and religion.

Professor Peirce was a transcendentalist in mathematics, as Agassiz was in zoology; and a certain subtile tie of affinity connected these two great men, however unlike they were in their special genius. Alike, also, they were in their enthusiasm which neither the piercing scepticism of Cambridge could wither, nor declining years chill with the frost of age. The thing he distrusted was routine and fanatical method, whether new or old; for thought, salient, vital, co-operative thought, in novel or in ancient aspects, he had nothing but respect and furtherance. Few men could suggest more while saying so little, or stimulate so much while communicating next to nothing that was tangible and comprehensible. The young man who would learn the true meaning of *apprehension* as distinct from *comprehension*, should have heard the professor lecture, after reciting to him. He was always willing to be esteemed for less than he had really accomplished; and he could join most heartily in the praise of others who even owed their impulse to him. *Modest* and *magnanimous*, but not unobservant, his ambition for personal distinction was early and easily satisfied; and he thus rid himself of what is to most men a perturbing, and too often an ignoble, element of discomfort.

Professor Peirce habitually ascribed to his listener a power of assimilation which the listener rarely possessed. He assumed his readers could follow wherever he led; and this made his lectures hard to follow, his books brief, difficult, and comprehensive. When, however, his listeners were students who had previously attained some skill as mathematicians and who had been trained in his own methods, the resulting work would be of *the highest order of excellence.* He was personally magnetic in his presence. His pupils loved and revered him; and to the young man, he always lent a helping hand in science. He inspired in them a love of truth for its own sake.

His own faith in Christianity had the simplicity of a child's; and whatever radiance could emanate from a character which combined the greatest intellectual attainment with the highest moral worth, that radiance cast its light upon those who were in his presence. "*Every portion of the material universe*," writes Professor Peirce, "*is pervaded by the same laws of mechanical action which are incorporated into the very constitution of the human mind.*" To him, then, the universe was made for the instruction of man. With this belief he approached the study of natural phenomena not in the spirit of a critic, but

reverently in the mood of a sympathizing reader; and the lesson he reads is: "*There is but one God, and science is the knowledge of Him.*" In his lectures and teaching he showed, as he always felt with adoring awe, that the mathematician enters (as none else can) into the intimate thought of God, sees things precisely as they are seen by the Infinite Mind, holds the scales and compasses with which the Eternal Wisdom built the earth and meted out the heavens. This consciousness had pervaded his whole scientific life. It was active in his early youth, as his coevals well remember; it gathered strength with his years; and it struck the ever recurring key-note in his latest public utterances.

Benjamin Peirce was a devout, God-fearing man; he was a Christian, in the whole aim, tenor, and habit of his life. To know Professor Peirce was simply to love him, to admire him, and to revere him. Since he was conversant with the phases of scientific infidelity, and by no means unfamiliar with the historic grounds of scepticism, it can not be regarded otherwise than with the profoundest significance, that a *mind* second to none in keen intuition, in aesthetic sensibility, in imaginative ferver, and in the capacity of close and cogent reasoning, *maintained* through life an unshaken belief and trust in the power, providence, and love of God, as beheld in his works, and as incarnate in our Lord and Savior. In one of his lectures on *Ideality in Science*, he said: "Judge the tree by its fruit." Is this magnificent display of ideality a human delusion? Or is it a divine record? The heavens and the earth have spoken to declare the glory of God. It is not a tale told by an idiot, signifying nothing. It is the poem of an infinite imagination, signifying immortality."

In May, 1880, Professor Peirce began to pass under the shadow of the cloud of his last illness. For some weeks there was little serious fear that it was a shadow not destined to lift. He was first confined to his chamber, on the 25th of June, 1880; and from that time, his slowly failing condition was hardly relieved even by any deceptive appearances of improvement. He died on the morning of Wednesday, October 6, 1880. Distinguished throughout his life by his freedom from the usual abhorrence of death, which he never permitted himself either to mourn when it came to others, or to dread for himself, he kept this characteristic temper to the end, through all the sad changes of his trying illness; and, two days before he ceased to breathe, it struggled into utterance in a few faintly-whispered words, which expressed and earnestly inculcated a cheerful and complete acceptance of the will of God with regard to him.

The funeral took place on Saturday, October 9, 1880, at Appleton Chapel, and was the occasion of an impressive gathering of people of great and various mark. The attendance included a very full representation of the various faculties and governing boards of the University; a large deputation of officers of the United States Coast and Geodetic Survey, headed by the superintendent and the chief assistant; delegations of eminent professors from Yale College and the Johns Hopkins University; many members of the class of 1829; and a great number of other friends of the deceased.

The pall-bearers were: President Charles W. Eliot; Ex-President Thomas Hill, Pastor of the First Parish Church, Portland, Maine; Capt. C. P. Patterson, Superintendent of the United States Coast Survey; Professor J. J. Sylvester, of the Johns Hopkins University; Hon. J. Ingersoll Bowditch; Professor Simon Newcomb, Superintendent of the American *Ephemeris and Nautical Almanac*; Dr. Oliver Wendell Holmes; Professor Joseph Lovering; and Dr. Morrill Wyman. A beautiful and simple service was conducted by the Rev. A. P. Peabody and the Rev. James Freeman Clarke.

In the career of Professor Benjamin Peirce, America has nothing to regret, but that it is now closed; while the American people have much to learn from his long, useful, and honorable life.

REMARKS ON SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

[Continued from May Number.]

Since $a.bbc=acb$ and $bc.\ ab=abc$ it follows that the result is not always independent of the order in which we perform the operations indicated by two substitutions. In the equation $ab.bc=acb$ we call ab and bc the *factors* and acb the *product* and the process is called *multiplication of substitutions.* The above example shows that the law that the product is independent of the order of the factors does not hold true with respect to the multiplication of substitutions.

The number of substitution groups increases very rapidly as the number of letters increases. During the last few years the work of making complete lists of such groups has been carried through ten letters* but no formula has yet been published by means of which the number of such groups can readily be determined for any number of letters.

If an expression involving a given number of letters is unchanged by applying all the substitutions of a group of the same number of letters to it but is changed by applying any other substitution of the same or a lower number

*The number of groups of ten letters exceed one thousand. Complete lists are found in the Quartly Journal of Mathematics as follows: Cayley: Substitution groups for two, three, four, five, six, seven, and eight letters, vol. 25, pp. 71-88, 137-155. Cole: List of substitution groups of nine letters, vol. 26, pp. 372 388. Cole: List of transitive substitution groups of ten and eleven letters, vol. 27, pp. 39-50. Miller: Intransitive groups of ten letters, vol. 27, pp 99–116.

A few errors and omissions with respect to these lists have been noted in late numbers of the Bulletin of the American Mathematical Society. The lists are complete in the sense that an effort is made to give all the possible groups of the given number of letters and fairly accurate results have been attained.